

Vehicle Automation and Freeway ‘Pipeline’ Capacity in the Context of Legal Standards of Care

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Abstract

The study evaluates, in the context of freeway segments, the interaction between automated cars' kinematic capabilities and the standard legal requirement for the operator of an automobile to not strike items that are in its path (known as the 'Assured Clear Distance Ahead' criterion). The objective is to characterize the impacts of ACDA-compliant driving behavior on the system-level indicator of roadway-network capacity.

We assess the barriers to automated cars operating non-ACDA-compliant driving strategies, develop a straightforward ACDA-compliant automated-driving model to analytically estimate freeway 'pipeline' capacity, compare this behavior to human drivers, and interpret quantitative findings which are based on a range of rationally-specified parameter values and explicitly account for kinematic uncertainty.

We demonstrate that automated cars pursuing ACDA-compliant driving strategies would have distinctive "fundamental diagrams" (relationships between speed and flow). Our results suggest that such automated-driving strategies (under a baseline set of assumptions) would sustain higher flow rates at free-flow speeds than human drivers, however at higher traffic volumes the rate of degradation in speed due to congestion would be steeper. ACDA-compliant automated cars also would have a higher level of maximum-achievable throughput, though the impact on maximum throughput at free-flow speed depends on the specific interpretation of ACDA. We also present a novel quantification of the tradeoff between freeway-capacity and various degrees of safety (one failure in 100,000 events, one failure in 1,000,000, etc.) that explicitly accounts for the irreducible uncertainty in emergency braking performance, by drawing on empirical distributions of braking distance testing. Finally, we assess the vulnerability of ACDA-compliant automated cars to lateral 'cut-ins' by vehicles making lane changes.

The paper concludes with a brief discussion of policy questions and research needs.

Keywords: Vehicle Automation, Freeway Capacity, Assured Clear Distance Ahead, Duty of Care

1. Introduction

With the commercial rollout of increasingly-sophisticated vehicle-automation concepts, there is growing interest in the nature of potential impacts to various aspects of the transportation system, such as safety (Fagnant and Kockelman 2014; Kalra and Paddock, 2016), parking (Zhang et al. 2015, Kong et al. 2016), pollutant emissions (Wadud et al. 2016), regulations (Smith 2014, Anderson et al. 2016 Gasser 2016), possible shifts from car-ownership to car-rental (Fagnant and Kockelman 2016), and how in-vehicle time is used (Malokin et al. 2015, Zmud et al. 2016).

This paper's focus relates to the impacts of highly-automated cars (cf. SAE 2014) on traffic flow on mainline freeway segments. We consider operating regimes in which the Automated Vehicle's (AV's) longitudinal-control system makes and then implements control decisions regarding speed and following-distance-from-forward-vehicle without input from the vehicle's occupant(s). In other words, this is a more highly automated concept than Adaptive Cruise Control (ACC), in which a human driver makes an initial selection of speed and following distance, and the automated control system subsequently operates pursuant to this guidance. The kinematics of these two concepts are not fundamentally different, though the lack of guidance from a responsible human driver leads to distinctive issues of responsibility for vehicle operation (see Section 5).

This topic has been addressed by a number of earlier studies (cf. Section 2); the original contributions of this research are as follows. First, 'regular' (i.e. non-emergency) freeway driving behavior is evaluated from the perspective of '*defensive driving*'; we document the current legal requirements for 'defensive driving' and subsequently employ them to specify and numerically test various plausible manifestations of this general approach. Earlier work (cf. Goodall 2014) has discussed the ethical issues of programming AVs' behavior for circumstances where crashes are imminent ("crashing algorithms"); we build on prior work by Carbaugh et al. (1998) and Kanaris (1997) in simulating car-following decision-making for mundane driving conditions which explicitly recognizes the potential for emergency conditions suddenly arising. Second, the kinematic parameters of the numerical analysis are derived from empirical field tests, and we take explicit account of uncertainty in braking-system performance to quantify the trade-off between crash-risk and capacity. We demonstrate that, under contemporary "*Rules of the Road*", faithfully replicating the behavior (see Section 4.5.2) of human drivers on a congested freeway could expose the manufacturers of automated-driving systems to a degree of liability¹ that they may find unacceptable, but reducing this risk would

¹ We note that LeValley (2013) argues that a manufacturer of highly-automated cars may be deemed to be a *common carrier*. Common carriers are generally required to exercise the "*highest degree of care*", a more stringent standard than the "*ordinary or reasonable care*" standard to which drivers are generally held.

unavoidably reduce the capacity of the freeway network. Designers of automated control algorithms might be viewed less sympathetically than individual human drivers and be more attractive targets for litigation (as they might be seen to plausibly be able to pay out large claims for both actual as well as punitive damages).

The analysis we report in this paper is subject to several important limitations, including that the discussion of legal issues is based solely on American (U.S.) law (the consequent numerical calculations, however, are not specific to any particular jurisdiction). A second limitation is that the analysis focuses exclusively on mainline freeway segments; weaving, merging and diverging behavior are all left as topics for future research. Third, we consider only automobiles (leaving as future research needs the unique challenges posed by heavy vehicles, recreational vehicles, motorcycles, etc.) and for purposes of the numerical analysis we treat the kinematic parameters of automated cars as identical for all automated cars, whereas different manufacturers' vehicle-control systems are likely to have distinctive properties (which are not currently knowable, due to both commercial confidentiality and the expectation of technological improvement in coming years). Fourth, the numerical analysis does not explicitly take account of the effects of uncertainty in automated vehicles' sensor readings. Fifth, we focus exclusively on traffic streams of homogenous AVs; we leave issues of mixed human and AV traffic (including non-homogenous capabilities of AVs) for future research.

Throughout this paper, we employ the term 'capacity' to refer to the maximum throughput (vehicles per lane per hour) that can be sustained at a given free-flow speed. For human-driven cars, the standard model of traffic flow (TRB 2010) suggests that this value (1,000 vehicles/lane/hour at a 75 miles/hour [121 km/hour] free-flow speed) is much lower than typical quoted values of freeway-lane capacity, as it refers to maximum throughput *at the free-flow speed*, whereas typical quoted values (e.g. 2,400 vehicles/hour/lane, per TRB 2010) refer to maximum throughput *across all speeds*, which in the case of a freeway facility with a 75 miles/hour [121 km/hour] free-flow speed is estimated by (TRB 2010) to occur at 53 miles/hour (85 km/hour) (see Section 4.2).

The remainder of this paper is organized as follows: Section 2 contains a background discussion of the relevant literature and legal standards, Section 3 introduces the methodology, Section 4 presents the numerical results, and Section 5 concludes the paper with a summary of findings and brief discussion of research needs.

2. Background

A number of earlier studies have evaluated the impacts of vehicle automation on network capacity. The subsequent discussion focuses on impacts of automation on freeway-network capacity; readers interested in studies of automation on arterial networks are directed to Dresner and Stone (2008), Ferreira and d'Orey (2012), Li et al. (2013), and Le Vine et al. (2015, 2016).

The distinct concepts of *connectivity* (V2X communication) and *automation* (control by algorithm rather than human driver) are in practice closely intertwined in the literature, with various studies

evaluating the interaction between them (cf. Kanaris et al. 1997, Vander Werf et al. 2002, Van Arem et al. 2006, Milanés and Shladover 2016, Motamedidehkordi et al. 2016, Zhou and Qu 2016, Asare and McGurrian 2016). For the purposes of this paper, however, the primary focus is on *unconnected automation*, in which each automated vehicle makes its own decisions on the basis of only line-of-sight information about its surroundings received from its own sensors. Whether other sources of information, specifically V2X communications, are received and disregarded (i.e. not considered trustworthy for decision-making purposes) – or simply not received – is treated as immaterial for the purposes of control decision-making. Whether AVs will be ‘self-directed’ or will rely on V2X communications to ‘direct their movement’ is noted by Glancy et al. (2016) as a major point of uncertainty.

In order to evaluate the impacts of automation on network capacity, a necessary starting point for comparison purposes is the existing set of models of human-driving behavior (Brackstone and McDonald, 1999). Noteworthy models include the *2010 Highway Capacity Manual’s* methodologies (hereafter referred to as “HCM-2010” (TRB 2010) and Wiedemann’s models of car-following behavior (Wiedemann 1974, 1999²), which are employed in the present paper, as well as models proposed by Gipps (1981), Yang and Kotsopoulos (1996), and Fritzsche (1994). Though the HCM-2010 model is the most widely used, none of these models is uniquely correct in describing the dynamics of traffic flow with human drivers; Brackstone and McDonald (2007, p.1183) report that “*providing clear unequivocal statements regarding car following and safety levels...is still far from straightforward*”. Indeed, we demonstrate a wide divergence between the HCM-2010 and Wiedemann-1999 models in their analysis of human-driven traffic flow at freeway speeds (see Section 4.2).

In analyzing the time gaps that human drivers leave behind the preceding vehicle, researchers have noted that humans regularly select time gaps that are much shorter than the values prescribed by standard advice (Brackstone et al. 2002; Friedrich 2016), with Brackstone (2002, p.43) remarking that human drivers appear to “*indulge in a certain amount of ‘educated risk taking’*”. The reasons for such behavior are poorly understood; hypotheses include signaling of ‘fitness’ in the context of evolutionary biology (Skippon et al. 2012), or drivers simply making rational choices to accept the small risk of striking a lead vehicle in exchange for the benefit of arriving at their destination very marginally more quickly. Despite the fact that individual human drivers routinely select small time gaps that are apparently ‘unsafe’ when analyzed against standard kinematic criteria, and regardless of human drivers’ reasons for doing so, it appears unlikely that vehicle manufacturers would do the same in the design of automated-control algorithms. The software code would leave little doubt about the decision-making logic (in contrast to acknowledged human frailties when making split-second decisions³), and the

² To the authors’ knowledge, there is no technical paper published by Wiedemann that documents the widely-applied “Wiedemann-1999” model; the model is described in PTV (2011).

³ Regarding acknowledgment of human frailties when making a split-second decision, Buchwalter et al. (2016 Negligence, Sect. 198) report that: “*Under the common-law ‘emergency doctrine,’ when an actor is faced with a sudden and unexpected circumstance which leaves little or no time for thought, deliberation,*

archived stream of information from an automated vehicle's on-board sensors would mean that the factual circumstances of a crash are also much less likely to be in dispute than in human-driver crashes for which the only evidence is physical and drivers'/eyewitnesses' memories.

Table 1 summarizes earlier studies that have evaluated the impacts on freeway capacity of automated vehicles operating with only line-of-sight information. Various methodologies have been employed, including both bespoke kinematic analysis and adaptation of car-following models of human-driving used in standard traffic-microsimulation software.

This study builds on a body of research originating in the 1990s into rear-end-collision warning and avoidance systems. Ervin et al. (1998) develop a simulation framework of adaptive cruise control (ACC) to investigate the empirical frequency of short-following-distance episodes and either human intervention or crash occurrence. Galler and Asher (1995) similarly present a control algorithm for incorporating V2V communications in automated car-following; both Ervin and colleagues and Galler/Asher focus on safety metrics rather than capacity impacts. Farber (1994,1995) reports similar studies, conducted as part of the REAMACS project. Carbaugh et al. (1998) evaluate the trade-offs between safety (defined by crash frequency and severity) and capacity on the basis of a requirement not to strike a leading vehicle. Carbaugh and colleagues evaluated kinematics in a similar manner to the present study, however they assumed emergency-braking by all vehicles at their maximum physically-achievable rate. By contrast, the present study considers a range of braking concepts for both the leading-vehicle/stationary-object (e.g. the 'weak' and 'strong' interpretations of the Assured Clear Distance Ahead doctrine described later in this Section) and following vehicle (e.g. occupant-comfort considerations, at the boundary between 'normal' and 'emergency' braking, and during wet driving conditions). As with the findings reported by Kanaris et al. (1997), Carbaugh and colleagues also do not quantify the trade-off between safety and capacity in the systematic manner (in order-of-magnitude increments up to a 1 in 1,000,000 risk) of the present paper, nor do these studies address the speed-flow relationship of AVs in congested-flow conditions (i.e. the fundamental diagram of traffic flow).

General limitations of earlier studies are that either: 1) the requirement to avoid striking a leading vehicle or previously-obscured object is not explicitly taken into account, or 2) deceleration concepts are specified in a narrower manner than in the present paper, or 3) properties of congested-flow conditions are not addressed.

Standard advice to human drivers is to "*drive defensively*"; this generally includes both anticipating unexpected behavior by other drivers and selecting an appropriate following distance behind a forward vehicle, though the specific advice offered varies both qualitatively and quantitatively in terms of what is a "safe" gap. New York, for instance, advises drivers to

or consideration, or causes the actor to be reasonably so disturbed that the actor must make a speedy decision without weighing alternative courses of conduct, the actor may not be negligent if the actions taken are reasonable and prudent in the emergency context...in certain situations, a person is not held to the strict standard of care required of a reasonably prudent person acting under ordinary circumstances."

select a two-second gap, whereas Florida advises four seconds (NYS DMV 2015, Florida DHSMV 2015). Texas also advises four seconds, and explicitly instructs motorists that “*you should always be able to stop within the distance you can see ahead of your car* (Texas DPS 2014, p.48).

The *Assured Clear Distance Ahead* doctrine is the common law manifestation of the *Defensive Driving* concept; it requires a vehicle operator to “*regulate his speed so that he can stop within the range of his vision*” (Pearson 2005, p.333)⁴. Buchwalter et al. (2016, Automobiles and Highway Traffic, Sect. 1115) report that “*Most jurisdictions follow the rule that a collision between a preceding and a following motorist gives rise to a presumption of negligence on the part of the following motorist, whether the preceding vehicle is moving, stopped, or stopping.*”

The *Sudden Emergency* doctrine qualifies the ACDA criterion; *Sudden Emergency* generally excuses a driver from negligence in the case of striking a vehicle, object, or person which has unexpectedly and suddenly moved laterally or vertically⁵ into their trajectory (Pearson 2005). For instance, “*A motorist driving at a reasonable speed and obeying the rules of the road is generally not liable for injuries to a child who darts in front of the vehicle so suddenly that the motorist cannot avoid injuring the child, as where a child darts out from behind other vehicles that were stopped in traffic, directly into the path of the vehicle, and there was no evidence that [the] driver was driving too fast*” (Buchwalter et al. 2016, Automobiles and Highway Traffic, Sect. 498). Similar logic has been found applicable in the circumstance of one vehicle abruptly changing lanes to place itself directly in front of another vehicle’s trajectory (Decker v. Wofford 1960), though the presence of stationary and reasonably expected road features (e.g. the crest of a hill limiting sight lines) has been held to not warrant application of the *Sudden Emergency* exception (Coppola v. Jameson 1972).

Glancy et al (2016) posit that AV manufacturers may face “*design-defect claims in which a particular programming choice associated with an accident is attacked as defective*” (p.39). Though ACDA is the generally-applied standard relevant to longitudinal vehicle control by human drivers, in practice it raises three major pragmatic issues relating to automated operation.

The first of these issues is that, notwithstanding their obligations under the ACDA rule, human drivers routinely violate it in the exercise of normal driving behavior. Leibowitz and colleagues (1998) document, for instance, that strict adherence to the ACDA criterion would require human

⁴ In some jurisdictions (e.g. France, cf. Fambro et al. 1997), there are examples of the assumed emergency maneuver used by road designers being a lateral swerve (steering) rather than emergency braking. The use of such criteria is clearly inappropriate in the estimation of freeway-segment capacity, however, because on a freeway operating at capacity it cannot be assumed that suddenly maneuvering laterally into an adjacent lane can be performed without resulting in a crash.

⁵ Buchwalter et al. (2016 Automobiles and Highway Traffic, Sect. 487) report that: “*A motorist who is proceeding along a highway where children are playing on plots of grass bordering it is under no duty to anticipate children dropping from trees.*”

drivers of automobiles to travel no faster than approximately 20 miles/ hour (32 km/hour) at nighttime on unlighted roads (regardless of roadway functional class), in order to be able to stop in time to avoid a *'dark-clad pedestrian'*. Despite traffic speeds at night exceeding 20 miles/hour (32 km/hour) on much of the unlighted road network, as well as posted speed limits far in excess of this speed, courts have attributed negligence to drivers not observing the ACDA rule at nighttime (cf. Mantz v. Continental Western Insurance Company 1988; Dranzo v. Winterhalter 1990). A fundamental tension arises from the fact that holding automated vehicles to the ACDA standard at all times could result in driving that is more conservative than human driving, which would tend to reduce capacity.

The second issue is that ACDA generally requires vehicle operators to avoid colliding with both the vehicle ahead (the 'weak' ACDA interpretation) as well as stationary objects (the 'strong' ACDA interpretation). If the following vehicle strictly observes the ACDA requirement to be able to "*stop within the range of his vision*", the vehicle must avoid striking a possible stationary object⁶ in the travel lane (e.g. a moderate-sized piece of road debris) which only becomes '*within the range of his vision*' once the leading vehicle (which blocks the forward line of sight) has passed over the foreign object. We show (see Section 4) that imposing this 'requirement on AVs would result in large decreases of roadway capacity as measured by vehicles-per-lane-per-hour.

The third pragmatic issue with rigid application of ACDA is that, even in cases where striking an object would not be negligent, the manufacturer may nevertheless wish to instruct the vehicle to drive more conservatively than the minimum legal standard. The designer may, for instance, plausibly view a heightened degree of occupant-safety as a marketing advantage. Under current law, the only apparent limitation to conservative driving behavior in most jurisdictions are statutes that prohibit 'impeding' the flow of traffic, and that such laws typically provide explicit exemption for slowing "when necessary for safe operation" (New York Vehicle and Traffic Law, Section 1181; cf. also California Vehicle Code Section, 22400a).

While courts have generally held that the operator of an automobile following another vehicle "*must handle the automobile in recognition of the superior rights of the traveler in front*" (Naffky 2012, Sect. 734), the operator of the following vehicle "*may assume that the preceding vehicle is being driven with care and caution, will use every precaution to avoid being struck in the rear by following vehicles, [and] will give timely warning, by an appropriate signal, of his or her purpose to stop, or decrease the speed of the car, or to turn*" (Naffky 2012, Sect.735). The obligation of the leading vehicle to signal when turning or braking has historically referred to visual indications, but may in the future be broadened to include electronic V2X messaging. In the case of visual turning/braking indicators, the leading vehicle's operator generally bears

⁶ In *Louisiana Farm Bureau Mutual Insurance v. Dunn (1986)*, for instance, the defendant was found negligent for having driven into a herd of cows that were standing on a state highway. However, the requirement to be able to stop for an unexpected stationary object is a very restrictive interpretation of ACDA and has not always been applied. Naffky (2012, Sect.664) reports, for instance, that "*A driver is not obligated to anticipate a body lying in the roadway, in the direct path of his vehicle; such an event constitutes a classic emergency situation, implicating the emergency doctrine.*"

responsibility for *signaling* their maneuver (Naffky 2012, Sect 698), rather than whether the following vehicle’s operator *receives and processes* the signal correctly (Naffky 2012, Sect 703). This is a minor distinction in the case of plainly-visible visual indicators (brake lights and turn signals), however in the case of V2X electronic messaging there is the novel possibility of data packets being successfully transmitted by the leading vehicle but failing to be received by the following vehicle (Bergenheim et al. 2014). A following vehicle would also quite reasonably place superior trust in the condition of its own sensing/processing equipment in comparison to the sensing/processing/V2V-signaling equipment of other vehicles on the road⁷. Whilst the forward driver owes a duty of care to the following vehicle (Naffky 2012, Sect. 734), the following vehicle’s lack of visibility beyond the vehicle ahead prevents it from knowing whether circumstances ahead of the forward vehicle have presented a sudden emergency that requires the forward vehicle to brake. Given that there is very little private benefit (in terms of travel time savings) to the occupants of a following vehicle from following closely behind the leading vehicle and potentially large costs (in the form of crash risk for which some combination of the following vehicle’s manufacturer and/or occupant would be liable), it appears prudent, under prevailing technology and rules of the road, for vehicles in a traffic stream to make control decisions on the basis of information from their own line-of-sight sensing.

3. Methodology

The numerical analysis of this study subsequently implemented ACDA-compliant driving behavior via a kinematic model. We then perform comparisons with capacity values calculated from the *HCM-2010* (TRB 2010) and *Wiedemann-1999* (PTV 2011) models of human driving behavior, and also draw on empirical “Naturalistic Driving” data from two sources (TRB 2013, USDOT 2016) to characterize additional dimensions of human-driving behavior. These tasks are described in this section.

Equations 1 and 2 present the minimum requirement for ACDA-compliant driving behavior, based on the fundamental equations of motion. Equations 1 and 2 are expressed in terms of headway (the time gap between the front of the leading vehicle and the following vehicle) and following distance, respectively:

$$H_{min} = t_{lag,f} + \frac{v}{2*a_f} + \frac{x_{veh} - \frac{v^2}{2*a_l}}{v} \quad (1)$$

$$x_{min} = v * t_{lag,f} + \frac{v^2}{2*a_f} - \frac{v^2}{2*a_l} + x_{veh} \quad (2)$$

⁷ The unique liability challenges posed by automated vehicle operating in platoons and other non-ACDA-compliant driving behavior that requires ‘trusting’ V2X messaging have been recognized for many years (cf. Euler 1990, IVHS America 1992, Khasnabis et al.1997, Kalra et al. 2009).

$$C_v = \frac{1}{H_{min}} \quad (3)$$

Notation is described in Table 2; the derivation of Equations 1 and 2 can be found in the Appendix of Le Vine et al (2016). Capacity (C_v) at free-flow speed v is the maximum throughput of vehicles at the specified speed; it is calculated by taking the reciprocal of the minimum headway value.

We note that a single value of velocity is specified for both veh_l and veh_f (a pair of leading and following vehicles) to represent both vehicles having the same initial speed under conditions of uncongested flow. Separate notation (a_l^- and a_f^-) is shown, however, for the rates of deceleration. These are the rates of deceleration that would govern in case of veh_l initiating emergency braking without advance warning and veh_f responding by also braking. We present results under various assumptions for a_l^- and a_f^- .

The sources of empirical parameter values for automated vehicles are presented in Table 3. A particularly important assumption is the value of $t_{lag,f}$: the maximum possible amount of elapsed time that veh_f assumes can take place between veh_l and veh_f initiating emergency braking. Shladover (1997) points out that it is “very difficult” (p.14) to select uniquely correct values for $t_{lag,f}$, with (Bierstedt et al. 2014, p.22) noting that “*the exact operating characteristics of automated vehicle technologies are proprietary and unknown*”. This is due to multiple interacting factors including commercial sensitivity on the part of automotive manufacturers, and improvement over time in sensing/processing/actuating capabilities. For the purposes of this study, we generally specify $t_{lag,f}$ to be 0.4 seconds, though we test (in *Scenario #8*) the effect of an arbitrarily-small latency value. The specification of 0.4 seconds is greater than the 0.2 seconds of latency described in Anderson et al. (2013), which is predicated on ideal lighting and weather conditions; we selected a larger value in order to account for the fact that an automated vehicle cannot know in advance with certainty whether or not lighting and environmental conditions will be optimal at the moment that circumstances abruptly change and emergency braking is required. A conservative amount of processing time also enables multiple sensor observations to be processed before initiating emergency braking. Limit false positives is an important consideration, particularly if the specified deceleration rate of emergency braking is sharp enough to be uncomfortable or dangerous for the AV’s unsuspecting occupants (e.g. if the deceleration could cause a hot beverage in the AV to spill onto an occupant).

We employ VISSIM software (PTV 2011) to implement the Wiedemann-1999 car-following model which we use (in addition to HCM-2010) as a descriptor of humans’ driving behavior on freeways. Where we report results from the Wiedemann-1999 model, they are on the basis of a 60-minute analysis period, which is preceded by a 15-minute ‘warm-up’ period. The network consisted of a single-lane freeway-class road segment five km (3 mi) in length, with 0% grade and no horizontal or vertical curves. All vehicles were defined to be ‘cars’, with identical ‘desired speeds’. Demand was specified (at 3,000 veh/hr) to be comfortably in excess of capacity for human drivers; capacity conditional on speed was calculated by measuring achieved flow for different ‘desired speeds’. The numerical results we present are averages across ten simulation runs for each desired speed.

3.1 Specification of scenarios

The analysis we present in Section 4 is based on a set of 11 scenarios, to account for uncertainties in AVs' prospective operational regimes. The scenarios were selected to expose various properties of the simulation system, and are intended to be representative rather than exhaustive.

In the *Baseline 'Weak' Scenario*, a_l^- and a_f^- are specified to be -28.3 ft/sec^2 (8.6 m/sec^2) and -16.4 ft/sec^2 (5.0 m/sec^2), respectively. This represents the 'weak' interpretation of the ACDA rule (as do all Scenarios other than the *Baseline 'Strong' Scenario*). -28.3 ft/sec^2 (8.6 m/sec^2) is the maximum observed sustained rate of human-driver deceleration of passenger cars on dry pavement reported in (Fambro et al. 1997). -16.4 ($0.5g$; 5.0 m/sec^2) is both the upper limit of deceleration permitted for "Full Speed Range Adaptive Cruise Control Systems" and the lower limit of required deceleration of "Forward Vehicle Collision Mitigation Systems", as specified by the International Standards Organization's (ISO) *Standards #22179 and #22839* (ISO 2009, 2013). In other words, -16.4 ft/sec^2 (5.0 m/sec^2) is a reasonable breakpoint between 'normal' and 'emergency' deceleration. These values (-28.3 and -16.4 ft/sec^2 [8.6 m/sec^2 and 5.0 m/sec^2] for a_l^- and a_f^- , respectively) were previously used as the 'Baseline' scenario in the companion study of AVs' queue discharge at signalized intersections (Le Vine et al. 2016).

The *Baseline 'Strong' Scenario* represents the 'strong' interpretation of the ACDA rule: veh_f selects its gap for following veh_l such that it (veh_f) is able to stop in the case of a stationary object in its path that only becomes visible once it has been passed by veh_l (i.e. veh_l physically blocked the sight line from veh_f to the object, until veh_l passes the object). This Scenario uses -28.3 ft/sec^2 (8.6 m/sec^2) as the value of a_f^- .

Scenario #1 is identical to the *Baseline 'Weak' Scenario*, with the exception that a_l^- is specified to be -21.3 ft/sec^2 (6.5 m/sec^2), which is the maximum sustained braking rate reported by Fambro et al. (1997) for a passenger car on wet pavement. Comparing this scenario to the *Baseline 'Weak'* scenario tests the difference between capacity under conditions of dry and wet pavement.

In *Scenario #2*, a_l^- is specified to be -41.6 ft/sec^2 (12.7 m/sec^2), which is the maximum reported rate of deceleration for performance sports cars (specifically the *Chevrolet Corvette* and *Porsche 911*) (Motor Trend 2011). In this scenario, veh_f makes a more-conservative assumption that the maximum possible rate of deceleration of veh_l is the braking performance of a sports car, rather than a typical passenger car.

Scenario #3 simulates a less-conservative assumption, in which veh_f assumes that it will brake at the same rate as veh_l , regardless of how sharply veh_l brakes. Scenario #3 provides no margin of error for veh_f , as it implies that in the event of veh_l initiating emergency braking at any rate up to its maximum physically-achievable rate veh_f would also be able to reliably brake at the same rate.

Scenario #4 is somewhat less aggressive than *Scenario #3*. In *Scenario #4* veh_f assumes that it will be able to, if required, brake at the rate of 28.3 ft/sec² (8.6 m/sec²) that is typical of passenger cars (as in *Scenario #3*), but the distinction is that veh_f makes the more-conservative assumption that veh_l could brake at the maximum rate of a representative sports car.

The numerical deceleration values discussed up to this point have assumed that the braking rates sourced from the literature are point values, rather than drawn from a distribution. The reality, however, is that braking rates cannot be known in advance with full certainty. To accommodate this fact, we draw on empirical testing of antilock braking systems (ABS) performed by the US's *National Highway Traffic Safety Administration*, for which the results from individual emergency-braking test runs of a passenger sedan (Chevrolet Malibu) on four different days of dry conditions are published (NHTSA 2000 Tables E-4, F-4, G-4, and H-4). Using the standard deviation (0.67 ft/sec² [0.20 m/sec²]) of the results from 34 emergency stops⁸ and an assumption that the results are distributed according to a standard normal distribution⁹, we developed estimates of various percentile points of the distribution. The gap-selection strategy in *Scenario #5* is that veh_f assumes that veh_l will brake no more sharply than the 99.9th percentile of the distribution of typical passenger car braking on dry pavement (-30.38 ft/sec² [9.26 m/sec²]), and that it (veh_f) will brake no more weakly than the 0.1th percentile of the same distribution (-26.21 ft/sec² [7.99 m/sec²]). Assuming conservatively¹⁰ that the braking rates of veh_l and veh_f are uncorrelated, the joint probability of these two events ($a_l^- \geq 99\%$; $a_f^- \leq 1\%$) occurring is 1 in 1,000,000 (i.e. 0.0001%). In other words, veh_f is willing to subject its occupants to braking at the maximum physical limits of the vehicle, but assumes conservatively that there may be idiosyncrasies that make the achieved braking rate unknown a priori.

In *Scenario #6*, veh_f assumes the standard -28.3 ft/sec² (8.6 m/sec²) value for a_l^- , but is unwilling to expose its (veh_f 's) occupants to deceleration greater than the maximum rate of

⁸ The study's authors classified test drivers' braking efforts into classes 'A', 'B', 'C', and 'D', with class 'A' being the largest amount of force applied to the brake pedal. For the purposes of the present paper, we removed all stops in class 'D' prior to performing statistical analysis. These were stops in which the driver did not meet both of the following criteria: at least 50 lbf (pound-force) of braking force within 0.2 seconds of brake application, and at least 100 lbf within 0.3 seconds. This eliminated 16 of the 50 stops, resulting in the dataset analyzed in the present study consisting of 34 stops. For the purposes of (NHTSA 2000), the authors of that study also 'accepted as valid' (Section 1-IV, paragraph a-4) stops in classes 'A', 'B', and 'C'.

⁹ We tested this assumption via the Shapiro-Wilk test; the null assumption that the data are normally distributed was supported (i.e. not rejected) at the 95% confidence level, though the test statistic ($p = 0.07$) was near to the critical value of $p = 0.05$.

¹⁰ This is a conservative assumption because a degradation in rates of deceleration, as might perhaps be caused by an oily road surface, would be expected to affect both veh_l and veh_f , which would result in positive correlation in their maximum physically-achievable braking rates.

High-Speed Rail (-1.8 ft/sec² [0.5 m/sec²], per CA-HSRA 2004). This Scenario therefore tests the consequence of AVs providing their occupants with a very high standard of ride quality, as characterized by maximum expected longitudinal deceleration; this Scenario is tested because some researchers (Smith 2012; Anderson et al. 2016; Gucwa 2014; Speiser et al. 2014) have suggested that automated vehicles may enable their occupants to devote their travel time to a similar degree of productivity/leisure as time spent on high-standard rail services.

Scenario #7 specifies the standard -28.3 ft/sec² (8.6 m/sec²) value for a_l^- , and we then calculated the value of a_f^- required so that a free-flow speed of 75 miles/hour (121 km/hour) results in the maximum calculation of capacity (automated vehicles per lane per hour). This is because, as we report in Section 4, we found that there is a strong negative relationship between free-flow speed and capacity.

Scenario #8 is identical to the *Baseline 'Weak' Scenario*, except that it tests the sensitivity of the capacity calculation to an assumption of $t_{lag,f} = 0$ (i.e. no time passes between veh_l and veh_f initiating emergency braking)¹¹. In this scenario we assume that veh_l provides vehicle-to-vehicle communications to veh_f immediately upon initiating emergency braking, that this messaging is also received and processed instantly by veh_f , and further that there is no delay at all while veh_f builds pressure in its braking system. This scenario, while implausible, was selected as the limiting case (i.e. none of these components of latency can be negative).

Finally, *Scenario #9* is identical to the *Baseline 'Weak' Scenario*, except that x_{veh} (the length of veh_l) is increased by 25%. In other words, this Scenario tests the sensitivity of the capacity calculations to the length of the vehicles in the traffic stream.

4. Results

In this section we present numerical results for mainline-segment freeway capacity (vehicles per lane per hour) in the presence of automated vehicles, on the basis of the criteria specified in Section 3.

4.1 Scenarios of ACDA-compliant automated car driving

Figure 1¹² shows calculated maximum throughput values for automated vehicles (*Baseline 'Weak' and Strong' Scenarios* and *Scenarios #1 through #9*), as well as comparisons to human driving behavior as characterized by the HCM-2010 and Wiedemann-1999 models. All curves

¹¹ Ahmed-Zaid et al. (2011) document the potential for latency values of under 0.1 seconds for transmissions and receipt of V2V messaging, including time delays to account for message authentication.

¹² Tables A1, A2 and A3 contain the data for the curves plotted in Figures 1, 2, and 4 respectively.

initially increase from the origin, which reflects the fact that at very low speeds throughput is determined primarily by speed and vehicle length, rather than spacing between vehicles.

At highway speeds (60+ miles per hour [97 km/hour] for the purposes of this discussion), we find that automated-car throughput decreases as speed increases (the only exceptions are *Scenario #3* and, to a lesser degree, *Scenario #7*, which are based on implausibly aggressive assumed patterns of deceleration). This negative relationship is consistent with the HCM-2010 model of human-driving, but not the Wiedemann-1999 model, which shows no relationship between speed and capacity in the range 40-70 miles/hour¹³ (64-113 km/hour). Compared against the HCM-2010 model of human drivers, for automated cars the rate of decrease (the slope of the curve) is smaller, demonstrating that the tradeoff between speed and throughput is less sharp for automated cars than HCM2010 predicts for human drivers (see also the discussion in Section 4.2 of Figure 2).

At highway speeds, the *Baseline 'Weak' Scenario* shows a complex set of findings regarding throughput. The calculated maximum level of throughput (vehicles/lane/hour) at a free-flow speed of 70 miles/hour (113 km/hour), for instance, of 1,893 for automated cars is larger than the HCM-2010 calculation (1,400 vehicles/lane/hour) but smaller than the Wiedemann-1999 simulated value of 2,470 vehicles/lane/hour. This is due to the divergence between the two human-driver models in their estimation of the relationship between throughput and speed at freeway speeds, with the Wiedemann-1999 model suggesting that free-flow speeds up to 70 miles/hour (113 km/hour) can be maintained at much higher volumes of traffic demand.

All plausible 'dry-pavement' Scenarios (i.e. excluding Scenario #1, which is discussed immediately below, and Scenario #8, which takes no account of latency $t_{lag,f}$) that result in greater throughput at 70 miles/hour (113 km/hour) than human drivers require that the occupants of automated vehicles be placed at risk of experiencing braking rates to avoid a rear-end crash that are near the passenger cars' maximum physically-attainable rates. In Scenario #6, where automated car occupants are not at risk of deceleration exceeding that of High Speed Rail, capacity at 70 miles/hour (113 km/hour) is 132 vehicles per lane per hour. This demonstrates the fundamental challenge to reliably delivering high-standard ride quality in road vehicles, even under automated control.

On wet pavement (Scenario #1), capacity is substantially higher (at all travel speeds) than on dry pavement if the ACDA criterion is interpreted to mean that an automated car must be able to stop to avoid striking the vehicle in front. This is consistent with findings in the context of ACDA-

¹³ At free-flow speeds above 70 miles/hour (113 km/hour), simulations using the Wiedemann-1999 model become unstable, resulting in large variations in throughput when using identical inputs with the exception of different random 'seed' values. As comparing the HCM-2010 and Wiedemann-1999 models at very high speeds is not the primary objective of this study, we restrict our discussion of the Wiedemann-1999 results to free-flow speeds up to 70 miles/hour (113 km/hour). We note the discrepancy of flow properties at very high speeds between these two models of human driver behavior as an item in need of further enquiry, to determine which better characterizes human driving behavior in this specific context.

compliant operations of automated cars at signalized intersections (Le Vine et al. 2016). This finding arises because on wet pavement veh_f can have confidence that veh_l will not be able to brake as harshly as it (veh_l) could on dry pavement. We note that assumptions underpinning this scenario are in contrast to an interpretation of ACDA to mean that a vehicle must be able to stop if a stationary previously-obscured object becomes visible, which is the subject of the Baseline ‘Strong’ Scenario; under this criterion capacity would be lower on wet pavement.

Comparing *Scenario #2* to the *Baseline ‘Weak’ Scenario*, it can be seen that there would be a substantial lessening of capacity if veh_f assumes that veh_l could brake at the rate of a performance sports car rather than a typical sedan-type automobile.

Scenario #5 is of particular interest, as it introduces uncertainty in physically-attainable rates of deceleration into the analysis. In this Scenario veh_f is willing to accept a specific 1 in 1,000,000 risk if veh_f initiates emergency braking (i.e. that a_l^- will not exceed the 99.9th percentile of the dry-pavement braking distribution, and that a_f^- will exceed the 0.1th percentile). The calculated value of automated-car capacity at 70 miles per hour (113 km/hour) is 4,217 vehicles per hour per lane – more than four times the capacity at 70 mph (113 km/hour) of human drivers, and a 76% gain relative to the 2,400 vehicles/lane/hour attainable by human drivers at any speed (this maximum occurs at 53 mph (85 km/hour) per the HCM-2010 model, see Section 4.2 below). We therefore conclude that the possibility of large gains in capacity exists even in the absence of cooperation between automated cars (i.e. without trusted, actionable V2V messaging), though this requires that the occupants of automated cars be at risk of sharp deceleration that is near to the physical limits of their vehicles.

Comparing *Scenario #8* to the *Baseline ‘Weak’ Scenario* shows the impact of the unattainable situation of zero latency ($t_{lag,f} = 0$); maximum throughput at 70 miles/hour (113 km/hour) is increased by 27%. *Scenario #9* shows that a large change (+25%) in vehicle length has a relatively marginal impact on automated car capacity at highway speeds (1,849 vehicles per lane per hour, versus 1,893 for the *Baseline Scenario*).

The *Baseline ‘Strong’ Scenario* shows (relative to the *Baseline ‘Weak’ Scenario*), the effect of the ‘strong’ interpretation of ACDA. As would be expected, this strict interpretation results in a lower calculated value of maximum throughput than the *Baseline Scenario* (which is based on the ‘weak’ ACDA interpretation), at all travel speeds (e.g. 1,501 versus 1,893 vehicles per lane per hour at 70 miles/hour [113 km/hour], a decrease of 21%).

4.2 Fundamental Diagram of ACDA-compliant automated car driving

The Fundamental Diagram is a standard method of displaying traffic flow properties, by showing any two of three quantities (speed, traffic density, and throughput¹⁴) plotted against one another (TRB 2010). Figure 2 shows the plot of speed versus throughput, using the parameters of the

¹⁴ If any two of these quantities are known, the third is also uniquely identified by the relationship that throughput equals the product of density and speed.

Baseline 'Weak' Scenario. Solid curves represent automated cars operating under constraints of various free-flow speeds (between 10 and 100 miles/hour [16 and 161 km/hour], in increments of 10), and dashed curves (derived from HCM-2010) show the analogous relationships for flow of human-driven cars on freeways with design speeds between 55 and 75 miles/hour (89 and 121 km/hour), in increments of 5 miles/hour (HCM-2010 displays only the curves shown here; no relationships are presented for freeways with free-flow speeds below 55 miles/hour [89 km/hour] or higher than 75 miles/hour [121 km/hour])¹⁵.

In Figure 2, the vertical portion of each curve of free-flow-speed (for both automated and human-driven cars) can be interpreted to mean that as traffic flow increases from zero, traffic will continue to flow at its free-flow speed until some critical level of density, beyond which a drop-off in speed occurs. Each automated-car curve (solid lines) is separate from other curves for its vertical portion (which represents the relevant floor at the free-flow speed of each curve), however the portions of the curves representing congested flow (the nonlinear sections) overlap one another as free-flow speed is found not to be relevant in this region of flow. Four phenomena are noteworthy when comparing the automated-car curves to the human-driver curves.

First, the transition from free-flow speed occurs at a lower level of throughput for human drivers; this is shown by the 'human-drivers' curves beginning to turn towards the left (from vertical, moving upwards from the x-axis) at lower throughput levels than automated cars.

Second, the transition from free-flow to constrained-flow is less abrupt for human drivers. This is reflected by the human-driver curves initially turning smoothly towards the left from their free-flow speeds, whereas there is a distinct 'kink' (formally, a discontinuity in the derivative) on each of the curves for automated cars. The relationship between Figure 1 and Figure 2 is that (for the *Baseline 'Weak' Scenario*) the throughput values in Figure 1 correspond to the throughput values of these 'kinks' in the Figure 2 free-flow-speed curves. This is also related to the relative steepness of the HCM-2010 curve in Figure 1; while the HCM-2010 model suggests that there is a stronger negative relationship between free-flow speed and capacity for human drivers than for automated cars, the drop-off of the HCM-2010 model from free-flow speed into congested speeds seen in Figure 2 initially has a very gentle slope.

The third notable phenomenon that can be seen in Figure 2 is that the maximum throughput value for automated cars (across all travel speeds; not limited to free-flow speed) is higher than for human drivers, even under the relatively conservative assumptions of the *Baseline 'Weak'*

¹⁵ Reported freeway capacity values have increased over time in the various editions of the *Highway Capacity Manual* that have been published since 1950. For instance, Laufer (2007) notes that maximum achievable throughput (irrespective of speed) is reported as 2,000, 2,200 and 2,400 vehicles/lane/hour in the 1985, 1994 and 2000 editions of the HCM, respectively. Laufer attributes these changes to improved traffic engineering, vehicle performance capabilities, and also possibly due to "changes in societal driving patterns" (p.3). Human driving behavior may therefore represent something of a moving target, which complicates using it as a benchmark to compare with automated control.

Scenario. For human drivers and automated cars, maximum throughput is 2,400 and 2,595 vehicles/lane/hour, respectively; this represents a gain of 8%.

Fourth, this maximum-achievable throughput of automated cars occurs at a lower speed (26 miles/hour) than for human cars (53 miles/hour [85 km/hour]). There is also a level of throughput (between approximately 2,000-2,250 and 2,400 vehicles/lane/hour, with the former of these values varying with human driver's free-flow speed) for which speed is calculated to be higher for human drivers than automated cars. This is represented visually by the region where the human-driver curves extend further to the right on Figure 2 than the automated-car curves. It is noteworthy that the finding of maximum throughput at lower speed is in contrast to results reported by Barth (1997); using the model developed by Barth it was found that maximum throughput for *platooning* automated cars occurred at higher speeds than for human-driven cars.

Stated succinctly, our results suggest that automated cars (under the *Baseline 'Weak' Scenario*) would be able to sustain higher flow rates at their free-flow speed than human drivers can achieve, but that the initial rate of degradation (the decrease in speed below free-flow speed caused by an incremental increase in traffic volume past this critical flow level) is larger for automated cars. Automated cars also have both a higher level of throughput at their free-flow speed than human drivers, and a higher level of maximum-achievable throughput (irrespective of speed).

4.3 Capacity calculations under uncertainty, with various thresholds of crash risk

Scenario #5 introduced uncertainty regarding achieved braking rates into the analysis, in recognition that actual deceleration rates for any given emergency braking event represent a draw from a distribution rather than a point value that is known *a priori* with certainty. *Scenario #5* examined the effects of veh_l planning for the possibility that it might be able to brake relatively weakly (at the 0.01th percentile of the distribution) while veh_f is able to brake relatively sharply (at the 99.9th percentile). In Table 4, we further investigate the effects of uncertainty in braking rates on capacity. This analysis takes account of both dimensions of uncertainty (in the value of both a_l^- and a_f^-) by drawing independently from two identical normal distributions with mean and standard deviation of 28.3 and 0.67 ft/sec² (8.6 and 0.20 m/sec²), respectively (i.e. $a_l^-, a_f^- \sim N(-28.3 \text{ ft/sec}^2, 0.67 \text{ ft/sec}^2)$). In Table 4 we examine both the 'weak' and 'strong' interpretations of ACDA, in which veh_f seeks to avoid striking veh_l if veh_l begins emergency braking (the 'weak' interpretation), or conversely veh_f seeks to avoid striking a stationary object that becomes visible to veh_f only after veh_l passes the object (the 'strong' interpretation).

For the analysis of the 'weak' interpretation, a total of 10 million independent realizations were taken from each of the two normal distributions, and for each pair of deceleration values (a_l^- and a_f^-) the minimum headway and concomitant capacity values were obtained. For the analysis of the 'strong' interpretation, only the 10 million realizations of a_f^- were used, as a_l^- is specified to be infinitely large. We note that this analysis accounts for only one specific source of uncertainty, of a type that can be readily characterized. Other plausible but less readily

characterizable sources of variation in braking system performance which would tend to ‘widen’ the empirical distribution of braking rates include, inter alia, vehicle make/model/age, brake temperatures, tire condition, and idiosyncratic road surface imperfections.

The results of this analysis are shown in Table 4. The first row demonstrates that if the designer of an automated car’s control algorithm selects the ‘weak’ interpretation of ACDA and wishes to accept no more than a one-in-a-million risk of a crash if the forward vehicle initiates emergency braking, the designer must specify a gap of no less than 0.69 seconds (on dry pavement, for traffic cruising at 70 miles/hour [113 km/hour]), which implies a maximum throughput at this speed of 4,108 vehicles/lane/hour. If the designer selects the ‘strong’ interpretation and the same crash risk, capacity is approximately 67% lower (1,367 vehicles/lane/hour).

Row #3 of Table 4 demonstrates that if the designer selects a somewhat less-conservative criterion (a one-in-ten-thousand risk of a rear-end crash, in the event of a_f^- initiating emergency braking), capacity at 70 miles/hour (113 km/hour) would increase by 8% (4,426 versus 4,108) and 2% relative to the ‘one-in-a-million’ risk threshold for the ‘weak’ and ‘strong’ interpretations, respectively.

The tenth row, to take a further example, shows that in order to achieve a capacity value at 70 miles/hour (113 km/hour) of 6,153 vehicles/lane/hour, automated cars would need to be programmed according to the ‘weak’ interpretation and also accept a 50% risk of striking the vehicle ahead in the event that the leading vehicle suddenly begins emergency braking. At the upper end of the distribution, the final row of Table 4 shows that capacity values in excess of 12,000 vehicles/lane/hour can in theory be achieved under the ‘weak’ interpretation, but this requires that each automated car runs a 999,999 out of 1,000,000 risk (i.e. a near-certainty) that it will strike the vehicle ahead if the leading vehicle initiates emergency braking. In general, throughput values for the ‘strong’ criterion are both much lower and much less variable across different possibility-of-crash values; the latter effect is due to there being only one degree of freedom (a_f^-), as a_l^- does not enter the analysis.

4.5 Comparisons to human driving

Drawing on empirical datasets, we now address two issues relating exclusively to human drivers, and a third relating to the interaction between human-driven and automated-control vehicles in mixed traffic:

- 1) *What is the frequency at which human drivers perform high rates of deceleration?* (Section 4.5.1)
- 2) *On congested freeways, do human drivers comply with the ACDA Rule?* (Section 4.5.2, and
- 3) *Is it likely that human drivers would ‘cut-in’ into the ACDA-compliant gap ahead of an automated car?* (Section 4.5.3)

4.5.1 Human drivers’ rates of deceleration

We address the first of these questions by drawing on the Naturalistic Driving Study (NDS) (TRB 2013), in which 3,645 participants at six regions across the U.S. had their private vehicles instrumented for extended periods during the 2010-2013 period (n=5.4 million journeys). Figure

3, prepared from tabulated data published by the NDS study team, shows the distribution of journeys by the maximum rate of deceleration experienced during each journey. The median journey experiences a maximum rate of deceleration between 9.7 and 12.9 ft/sec² (3.0 and 3.9 m/sec²), though there is a long lower tail. Roughly one journey in a thousand (i.e. 0.1%) experiences deceleration greater than 29.9 ft/sec² (9.1 m/sec²). Given that a one in a thousand risk is likely to be far too frequent to be selected as a crash-risk criterion, the NDS data appear to support the *Baseline 'Weak' Scenario* specification of automated cars assuming that the vehicle immediately ahead could unexpectedly brake at or near the vehicle's physical limits.

4.5.2 Human drivers' degree of compliance with the ACDA Rule

We addressed the second of these questions through analysis of disaggregate vehicle-trajectory data from the Next Generation Simulation (NGSIM) study (USDOT 2016). The NGSIM data were collected via video observation of three sites in California. We employ vehicle-trajectory data from the US 101 freeway site, which was collected from 7:50 to 8:35 AM on 6/15/2005; during this period traffic flow transitioned from uncongested to congested conditions. Each vehicle's location and velocity are observed at 0.1 second intervals. After removing all heavy vehicles and motorcycles (which have distinctive deceleration profiles), and all instances when an automobile was following either a heavy vehicle or a motorcycle, we applied Equation 2 to each automobile at each time-step, using various sets of parameter values as shown in Table 5. Table 5 shows that, even under implausibly aggressive assumptions about reaction time and rates of deceleration, human drivers on US Route 101 regularly violate the ACDA criterion. For instance, Scenario *NGSIM-9* shows that even if a human driver is prepared to brake at its vehicle's maximum physically-attainable rate (with -28.3 ft/sec² [8.6 m/sec²] specified as this value) and the driver assumes zero reaction time, human drivers are in violation of the ACDA rule 0.2% of the time under these assumptions. Under more-plausible assumptions (see, e.g., *Scenarios NGSIM-11* through *NGSIM-14*, shown in Table 5), human drivers can be inferred to have violated ACDA between 1.5% and 49% of the time. It appears that while the ACDA criterion is the standard to which human drivers are held for legal purposes, in practice human drivers regularly violate it on congested freeways.

4.5.3 Potential for human-driver cut-ins

The third question arises as to whether the headways required for automated cars to be ACDA compliant would result in human drivers unilaterally performing 'cut-in' maneuvers in heavy traffic, by changing lanes into the assured clear distance of an AV (cf. Milanés and Shladover (2016) in the context of platoons of CACC-equipped cars). In order to address this question, we compared the spacing that ACDA-compliant AVs would leave ahead of themselves (see Figure 4) with empirical evidence on the size of gaps accepted by human drivers in congested traffic, using the NGSIM U.S. 101 dataset (see Table 6). Table 6 shows average spacing (distance between the rear of veh_l and the front of veh_f), at the point in time when a third vehicle changes lanes to place itself between veh_l and veh_f . The average values range from 109 to 151 feet (33 to 46 meters), with average speeds in the range of 18 to 26 miles/hour (29 to 42 km/hour). At this speed range, the spacing values of AVs are less than 50 feet (15 meters) for all Scenarios, as seen in this Table A3 (except the implausible *Scenario #7* in which the automated vehicle's occupants have a ride quality similar to High Speed Rail). Therefore, on the basis of automated cars' ACDA-compliant spacing being no more than approximately

half of the average spacing at which human drivers perform lane changes, we conclude that ACDA-compliant AVs appear to not be vulnerable to the average ‘cut-in’ maneuver by a human driver at speeds up to 25 miles/hour (40 km/hour). This accounts for the *average* spacing of humans’ ‘cut-in’ maneuvers; further research is needed to evaluate the distribution of these spacing values in order to state with greater credibility how vulnerable ACDA-compliant vehicles would be to relatively aggressive ‘cut-in’ maneuvers by human drivers.

5. Conclusions

This paper presents a novel exposition of the *Assured Clear Distance Ahead* criterion associated with negligence in rear-end car crashes, followed by a numerical analysis of the impacts of ACDA-compliant AVs on freeway mainline capacity. Our focus is primarily on unconnected AVs, i.e. in which each vehicle maintains the primary responsibility for its actions. A set of scenarios was evaluated and compared to human drivers, in the interest of exposing the general properties of flow of ACDA-compliant AVs. We present a novel investigation of the consequences of risk-averse car-following behavior which explicitly accounts for uncertainty in braking system performance.

This research highlights a fundamental tension, namely that human drivers are in principle held to the ACDA standard but routinely violate it in practice, and this ‘negligent’ driving tends to increase traffic-moving capacity (i.e. to lower congestion). We demonstrate that, even if ACDA is deemed to be applicable for AVs as it is for human drivers and V2V signaling is deemed unreliable for the purposes of selecting following distances, large gains in freeway capacity are nevertheless possible. A requirement, however, is that the occupants of automated cars must be placed at risk of deceleration at or near their vehicle’s maximum physically attainable rate, which places limits on the cabin configuration of automated cars and the types of activities that their occupants can perform while traveling. While V2V signaling could in principle allow even greater capacity, this would require a level of trust in others that is not reflected in current rules of the road¹⁶.

An additional tension is that cars following one another closely on a freeway provides benefits mainly to upstream vehicles, in the form of decreased congestion. The occupants of the vehicle performing the close following, however, receive little in the form of direct benefit (time savings) and bear a higher risk of being held liable for striking the vehicle ahead in the traffic stream. Circumstances in which there is a systematic mismatch between the distribution of benefits and

¹⁶ Naffky (2012, Sect. 648) reports that: “A motorist may assume that other travelers will obey the rules of the road and act on that assumption, at least in the absence of anything that in the exercise of reasonable care would put him on notice to the contrary. According to other authority, a driver may ‘presume’ that other travelers will obey the rules of the road [such as advance signaling of maneuvers], but cannot ‘assume’ that they will since no person has a right to anticipate due care of obedience to the law on the part of others...circumstances may arise where a temporary violation thereof [of motor vehicle law] is excusable or even imperative”. See also footnote #20.

costs to public and private actors can result in suboptimal outcomes, however; this is a classic *collective action* problem (Ostrom 2015).

We now conclude this paper with a brief discussion of research needs for the next phase of the research agenda.

First, this research highlights the need for a program of testing the emergency braking capabilities of AVs. There is also a need to confirm through field testing whether or not the kinematic parameters employed in this paper will continue to be valid. The empirical degree of error-correlation between the braking rates of a leading and a following vehicle is a further issue in need of further enquiry; this study's assumption of independence (see Section 4.3) is likely to be conservative. Given that the consequences of unconnected automation could include substantial gains or losses of capacity on the publicly-owned freeway network, there appears to be a case for public dissemination of the full technical results, which would of course need to be balanced against the commercial interests of AVs' manufacturers.

Second, further research is needed into the traffic flow properties of ACDA-compliant AVs in congested conditions. The analysis presented here does not account for possible 'shockwave' phenomena in traffic streams of automated vehicles, which is complex for two reasons. The 'shockwave' phenomenon in human-driven traffic streams is itself neither "*well-understood*" nor "*accurately modeled in existing models*" (Yeo and Skarbadonis 2009, both quotes from p.99). Different manufacturers' AVs will also likely operate in idiosyncratic ways, though for reasons of commercial sensitivity and the absence of large fleet sizes (100s of vehicles) available for on-road testing, there is limited evidence today on which to base reliable conclusions. Therefore, researchers using simulation methods would need to make operational assumptions of parameters for which much less knowledge currently exists in the public domain, in comparison to the set of assumptions made in the kinematic model presented in this paper.

Third, the ACDA rule is not the only plausible standard for 'reasonable' operation of automated vehicles; research is needed into both the various interpretations of ACDA ('strong' versus 'weak') and alternatives to ACDA. For instance, a government may reach a considered conclusion that the benefits from increased capacity (and avoided costs from postponing/canceling planned road-widening projects) of non-ACDA-compliant operation of automated vehicles outweigh the marginal costs of a heightened risk profile, and may therefore plausibly define manufacturers' duty of care to permit (or require) more aggressive driving behavior than ACDA permits¹⁷. Glancy et al. (2016) note the possibility of 'safe harbor' legislation or regulation that could provide manufacturers a defense against liability on the basis of satisfying a safety standard set by policymakers. Even if such policies are enacted by neither legislation nor regulation, Anderson et al. (2016) note that it is likely that AV manufacturers will

¹⁷ Anderson et al. (2016, p.142) report that "*International air travel, nuclear power, and vaccines are all areas in which a promising technology received the subsidy of liability protection*", but subsequently argue that "*even if AV [automated vehicle] technology creates considerable positive externalities, it is not clear that altering the tort system is the best way to subsidize it*".

argue in court that the ‘reasonableness’ standard in allocating liability should consider AVs’ societal cost-benefit analysis (which the authors expect to be demonstrably favorable), whereas plaintiffs in AV crashes may by contrast be incentivized to argue that ‘reasonableness’ is to be determined on the basis of a narrower cost-benefit analysis limited to the specific component of the AV-driving algorithm that governed the vehicle’s behavior proximate to the crash. If, however, the ACDA criterion as it is applied to human drivers is treated as the relevant criterion for AVs’ operation, guidance will be needed regarding whether AVs driving on limited-access freeways may/should/must interpret the ACDA criterion ‘weakly’ or ‘strongly’. Also, any departure from the ACDA criterion in the interest of maximizing capacity (or in locations where it cannot be guaranteed, such as on weaving segments of freeways) would require in-depth analysis to determine whether chain-reaction collisions can be avoided when an initial crash occurs in a traffic stream of vehicles that are not operating in ACDA compliance and, if not then how the costs of the crash are allocated¹⁸. Shladover (1997) studied the problem of sub-ACDA-compliant spacing, highlighting the fact that in AV platoons with very close headways any crashes would be relatively low-energy impacts (because there is little distance for differential rates of deceleration to lead to large speed differences), however in the process of longitudinally forming a platoon there is unavoidably a point at which ACDA compliance is impossible but the headway has not yet become small enough to ensure a low-energy crash.

Fourth, there is a need to quantitatively establish the safety/efficiency tradeoffs in the context of AVs’ control. The fact that kinematic parameters cannot be known *a priori* with certainty means that there is no combination of car-following distance and speed that could completely eliminate the possibility of striking the leading vehicle. Therefore, research that rigorously exposes, in numerical terms, the specific safety/efficiency tradeoffs (extending from the initial analyses that we report in Section 4.3) would enable manufacturers to design algorithms for which it can be demonstrated that the anticipated benefits exceed the anticipated costs (cf. the discussion in LaValley [2013] of the precedent established in *United States v Carroll Towing* [1947]¹⁹ to determine whether a party has performed an adequate duty of care).

¹⁸ Naffky (2012, p.129-131) reports that “*Ordinarily, where a chain of events begins, due to the negligence of the owner or driver of an automobile, he may be liable for all mishaps that are properly the proximate result of his improper conduct...In cases involving successive car accidents, proximate cause is resolved as a matter of law based on the following considerations: (1) lapse of time; (2) whether the force initiated by the original wrongdoer continued in active operation up to the injury; (3) whether the act of the intervenor [an intervening cause is a new and independent force that breaks the causal connection between the original wrong and the injury] can be considered extraordinary; and (4) whether the intervening act was a normal response to the situation created by the wrongdoer.*”

¹⁹ In (*United States v. Carroll Towing* 1947), it was held that duty of care is a function of three variables: the probability P of a harm occurring, the loss L to the person who is owed the duty of care if the harm occurs, and the burden B (or cost) of preventing the harm. The rule would accept that one’s duty is to incur costs for which the cost is less than the expected loss: $B < PL$.

Fifth, the fact that maximum throughput for AVs is calculated to occur at much lower speed (26 miles/hour [42 km/hour]) than for human drivers is a novel speed/capacity tradeoff that requires further research. This tradeoff is an undesirable system-level property because it introduces a new type of tension between two of the principal objectives of the transportation network.

Sixth, there is a general need to establish the circumstances under which V2X messages can be regarded as trustworthy and complete, and hence actionable for safety-critical decision-making. For instance, it is unclear at present how liability would be allocated in case of the failure of a leading vehicle's V2V signaling to be received and processed by a following vehicle²⁰.

Seventh, research is required to establish which person or entity is responsible in the event of non-ACDA-compliant driving by an AV, and how this may depend on circumstance. In circumstances where a human driver has made an informed selection of a speed and headway combination that is not ACDA-compliant and then instructed their vehicle to mimic this behavior in automated mode, it may be that the human driver bears some degree of responsibility in the event of a rear-end crash while in automated-driving mode. In this context, it is noteworthy that at least one designer of automated vehicle control algorithms (Urmson et al. 2015) has proposed a 'learning period' during which a human driver drives their automated car, while the vehicle 'learns' the human's driving style which it subsequently aims to mimic. In such a set of circumstances, the designer of the control algorithm might assert that the end user is at least partly liable in case of a rear-end crash while in automated mode, by claiming that the end user gave instructions to not follow the ACDA rule by not himself/herself having driven in an ACDA-compliant manner during the learning period. If, however, the owner/occupant of an automated car has never given instructions (or anything that can be construed as 'instructions') to an automated car regarding driving style, it may be more likely that liability for failing to comply with ACDA would rest with the manufacturer.

Eighth, this research is intended to provide support for transportation planners simulating the effects of AVs in regional travel-demand models. Therefore, there is a need to adapt the volume-flow relationships we report here (e.g. Figure 2) into curves analogous to the volume-delay curves employed in travel demand models. This research would ideally involve algorithms that account for spillback during periods of oversaturated demand.

From the earliest days of vehicle automation, it has been recognized that *"the most effective (and proactive) method to control costs related to liability is through careful product design and careful system operation"* (Stevens 1997, p.344). It is the authors' hope that this research will

²⁰ Regarding the requirements for maintain a vehicle generally and "signaling devices" specifically, Naffky (2012, Sections 621, 623, 668) reports: *"The duty to exercise due care in the operation of a motor vehicle by use of signals and warnings ordinarily carries with it a duty to have the vehicle equipped with a horn, bell, or other signaling device...[However,] lack of warning equipment in working order will not constitute actionable negligence unless such lack proximately causes an injury...[Further,] regulations as to signals, whether statutory or otherwise, call for the minimum of care and not the maximum."*

assist researchers, designers and policymakers in understanding the degree of care required in automated cars' operation, and the attendant consequences for traffic flow.

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Table 1: Summary of previous studies of the impacts of unconnected vehicle automation on freeway network capacity

Citation	Methodology	Specification of parameter values	Notable findings (relating to <i>autonomous</i> operating mode)
Kanaris et al. (1997)	Kinematic analysis (synthetic data)	Speed: 60 miles/hour (leading vehicle), 63 miles/hour (following vehicle); a_l : 0.8g; a_f : 0.72g; t_{lag} : 0.5 sec (Further parameters, e.g. constraints on 'jerk', can be found in Kanaris et al. 1997, p.147)	Headway between automobiles of 0.66 seconds
Carbaugh et al. (1998)	Kinematic analysis (empirical vehicle-trajectory data)	Latency of 0.3 seconds between leading and following vehicles braking; deceleration rates per empirical data approximated by Gaussian distribution with mean 7.01 m/s ² , std.dev 1.01 m/s ² , and clipped at 4 m/c ² and 10 m/s ²	At a flow rate of 2,500 veh/lane/ hour, 87-97% reduction in rear-end crash occurrence relative to human drivers in 1600 veh/lane/hr conditions
Malone and van Arem (2004)	Traffic microsimulation applying Burnham et al. (1974) car-following model with parameters adapted to automated vehicles	Time gap: 1.4 seconds	Throughput of 2,200 vehicles per lane per hour, where vehicles operate with adaptive cruise control
Asare and McGurrin (2016)	Traffic microsimulation, applying Wiedemann-1999 car-following model with parameters adapted to automated vehicles	Parameter descriptions can be found in Asare and McGurrin (2016), p.7 Look ahead distance: 850 feet; Temporary lack of attention: 0 seconds; Number of observed vehicles: 4; CC1: 0.8 seconds; CC2: 11.12 feet; CC3: -6 seconds; CC4: -0.2; CC5: 0.2; CC6: 10.44	Throughput of approximately 1,800 'autonomous vehicles' per lane per hour
Motamedidehkordi et al. (2016)	Traffic microsimulation, applying Wiedemann-1999 car-following model with parameters adapted to automated vehicles	Parameter descriptions can be found in Motamedidehkordi et al. (2016), p.6 CC0: 1 meter; CC1: 0.5 seconds; CC2: 4 meters; CC3: -8 seconds; CC4: -0.1; CC5: 0.1; CC6: 0; CC7: 0.25 m/sec ² ; CC7: 0.25 m/sec ² ; CC8: 3.5 m/sec ² ; CC9: 1.5 m/sec ²	Mean time headway for 'highly-automated vehicle' of approximately 0.85 seconds
Friedrich (2016)	Minimum headway calculated on basis of speed, vehicle length, minimum 'safety distance to vehicle ahead', and time gap	Speed: 70 km/hour; Vehicle length: 4.5 meters; Minimum safety distance: 3 meters; Time gap: 0.5 seconds	Capacity per lane of 3,900 automobiles per hour with "purely autonomous traffic" (p.326)

Table 2: Summary of notation (adapted from Le Vine et al. 2016)

Notation	Description
veh_l, veh_f	veh_l and veh_f denote the <i>leading</i> and <i>following</i> vehicles, respectively
H_{min}, x_{min}	H_{min} is the minimum headway (in units of time) between the rear bumpers of veh_l and veh_f that is consistent with the ACDA criterion. x_{min} is identical, but denotes spacing (units of distance) rather than time
x_{veh}	Length (units of distance) of veh_l
v	Free-flow speed of the traffic stream (assumed, without loss of generality, to be equal for veh_l and veh_f)
a_l^-, a_f^-	a_l^- and a_f^- denote the maximum rate of deceleration of veh_l and veh_f , respectively
$t_{lag,f}$	Maximum duration of time that the controller of veh_f is prepared to assume can elapse between veh_l commencing emergency braking and veh_f subsequently initiating emergency braking

Table 3: Values specified for each operational parameter for automated vehicle control

Parameter	Value (employed in all Scenarios except where otherwise indicated)	Notes
x_{veh}	19 feet (5.8 meters)	19 feet (5.8 meters) is the standard length of 'passenger car' design vehicle, per (AASHTO, 2011)
a_f^-	16.4 ft/sec ² (5.0 m/sec ²)	See discussion in Section 3.1
a_l^-	28.3 ft/sec ² (8.6 m/sec ²)	See discussion in Section 3.1
$t_{lag,f}$	0.4 seconds	See discussion in Section 3.1. Note that low-latency assumptions (e.g. 0.2 seconds) are predicated on ideal conditions of good lighting, fair weather and 0% failure rates in sensing/control (Anderson et al., 2013)
v	Varies	Results are reported for values of velocity between 1 and 100 miles/hour (161 km/hour)

Table 4: Simulated maximum throughput values based on minimum calculated time gap (rear of veh_l to front of veh_f) if veh_l and veh_f both attempt to brake at their physical maximum rate, using 10,000,000 random draws from independent normal distributions, on dry pavement with $v = 70$ miles/hour (113 km/hour) and $t_{lag} = 0.4$ sec

A	B	C	D	E
	Criterion is for veh_f to avoid striking veh_l if veh_l unexpectedly initiates emergency braking		Criterion is for veh_f to avoid striking a stationary object that becomes visible to veh_f only after veh_l passes the object	
Probability of crash	Minimum gap (seconds), conditional on veh_f accepting the crash risk in Column 'A'	Maximum throughput (veh/lane/hour at $v = 70$ miles/hour [113 km/hour]), based on minimum headway shown in Column 'B'	Minimum gap (seconds), conditional on veh_f accepting the crash risk in Column 'A'	Maximum throughput (veh/lane/hour at $v = 70$ miles/hour [113 km/hour]), based on minimum headway shown in Column 'D'
0.0001%	0.69	4,108	2.44	1,367
0.001%	0.66	4,247	2.41	1,383
0.01%	0.62	4,426	2.38	1,399
0.1%	0.58	4,653	2.35	1,416
1%	0.54	4,953	2.31	1,437
2.5%	0.51	5,111	2.30	1,447
5%	0.50	5,255	2.28	1,456
10%	0.47	5,431	2.27	1,466
25%	0.44	5,751	2.24	1,482
50%	0.40	6,153	2.21	1,501
75%	0.35	6,616	2.18	1,519
90%	0.32	7,089	2.16	1,535
95%	0.30	7,423	2.14	1,544
97.5%	0.28	7,730	2.13	1,553
99%	0.25	8,123	2.11	1,562
99.9%	0.21	9,094	2.09	1,582
99.99%	0.17	10,099	2.06	1,598
99.999%	0.13	11,181	2.04	1,613
99.9999%	0.10	12,283	2.02	1,626

Table 5: Percentage of time that human drivers on a congested freeway are in violation of the ACDA Rule, using the U.S. 101 NGSIM dataset

Scenario	a_l^- (ft/sec ²)	a_f^- (ft/sec ²)	$t_{lag,f}$ (seconds)	Percentage of time that human drivers are in violation of the ACDA Rule
NGSIM-1	28.3 (8.6 m/sec ²)	16.4 (5.0 m/sec ²)	0	1.8%
NGSIM-2	28.3	16.4	0.1	2.9%
NGSIM-3	28.3	16.4	0.5	12.5%
NGSIM-4	28.3	16.4	0.7	20.3%
NGSIM-5	28.3	16.4	1	33.7%
NGSIM-6	28.3	16.4	1.75	63.5%
NGSIM-7	28.3	16.4	2.5	80.3%
NGSIM-8	28.3	16.4	3.5	90.0%
NGSIM-9	28.3	28.3	0	0.2%
NGSIM-10	28.3	28.3	0.1	0.3%
NGSIM-11	28.3	28.3	0.5	1.5%
NGSIM-12	28.3	28.3	0.7	4.4%
NGSIM-13	28.3	28.3	1	13.9%
NGSIM-14	28.3	28.3	1.75	49.3%
NGSIM-15	28.3	28.3	2.5	74.0%
NGSIM-16	28.3	28.3	3.5	88.0%

Note: values of $t_{lag,f}$ are sourced as follows: 0 is the lower bound; 0.1 is a near-zero value ; 0.5 and 0.7 seconds are the lowest value of human-driver latency time before braking in response to anticipated and unexpected objects (respectively) consistently observed per Table C-3; 1.0 second is the approximate mean latency time, per Table C-5; 1.75 seconds is the 99th percentile latency time for unexpected objects per Table C-6; 2.5 and 3.5 seconds are the 95th and 99th percentile latency time for unexpected objects when observation by the researcher is covert, per Table C-6; all references in this note refer to (Fambro et al. 1997).

Table 6: Size (feet) of gaps selected by human drivers when changing lanes on a congested freeway, from the U.S. 101 NGSIM dataset (Cambridge Systematics, 2005)

Time period	Average speed of traffic (space mean speed, miles/hour)	Average forward spacing (feet) of vehicles that changed lanes, immediately following the lane change	Average rear spacing (feet) of vehicles that changed lanes, immediately following the lane change	Average overall spacing (feet) immediately prior to lane change, including an assumed vehicle length of 19 feet
7:50 AM to 8:05 AM	25.7	63.4	69.0	151.4
8:05 to 8:20	21.6	46.6	56.6	122.2
8:20 to 8:35	18.0	41.1	48.6	108.7

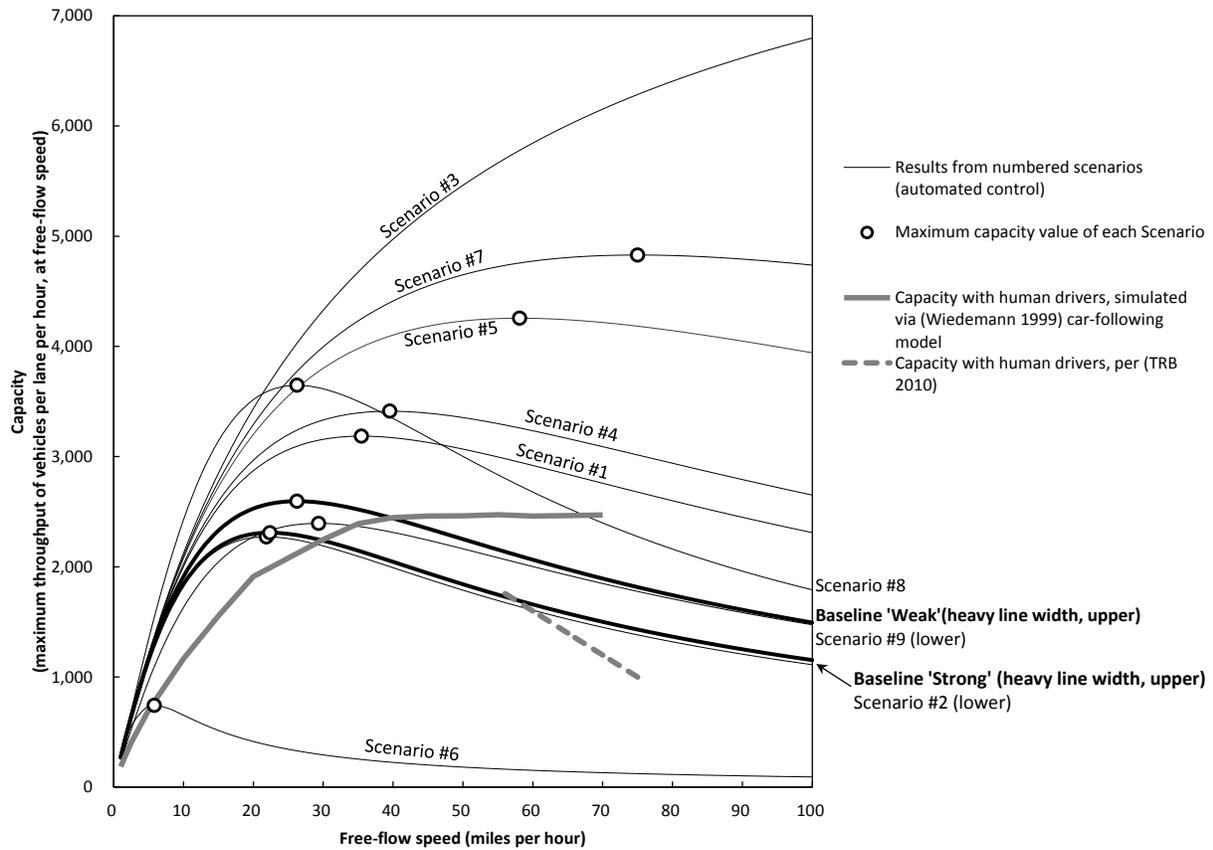


Figure 1: Calculations of capacity for automated cars (Baseline 'Weak' and 'Strong' Scenarios and Scenarios #1 through #9), with comparisons to human-driving behavior (HCM-2010 and Wiedemann-1999).

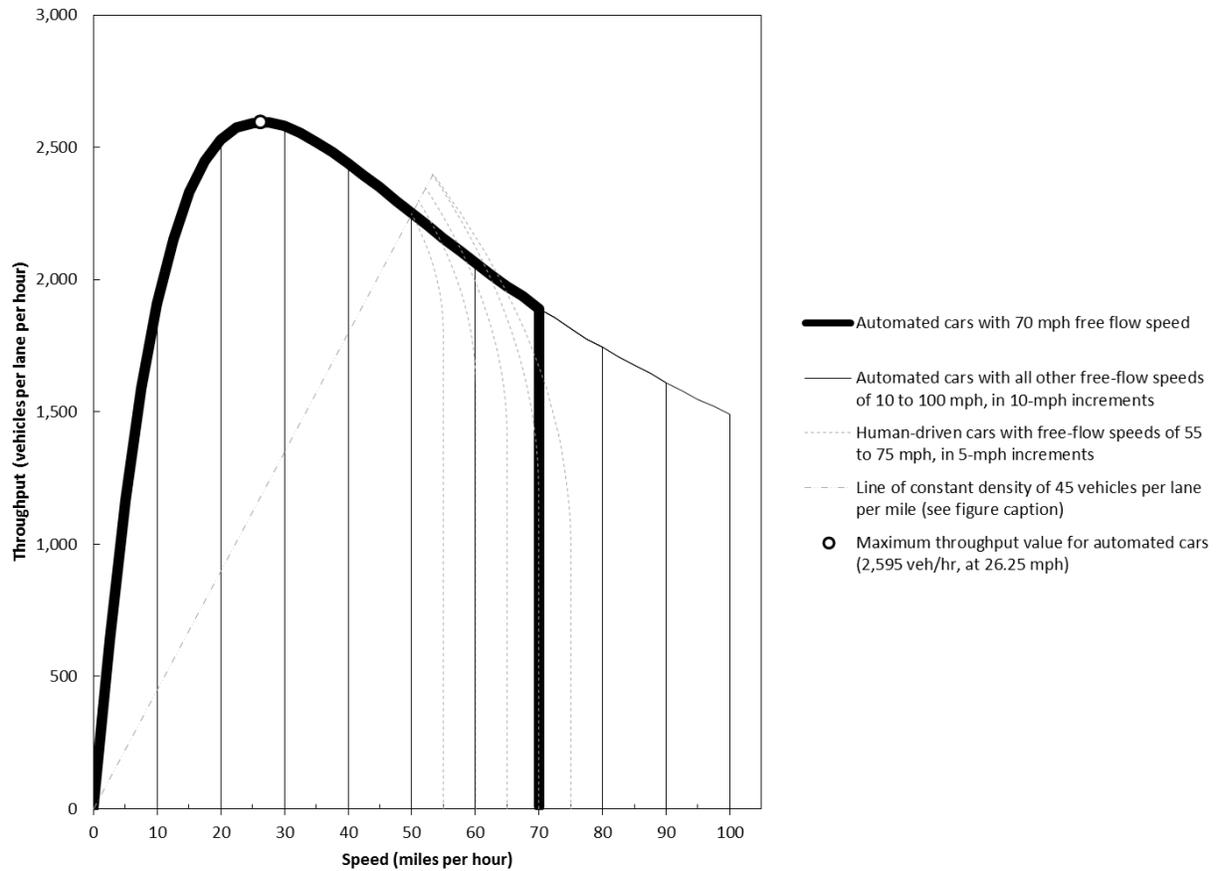


Figure 2: Speed versus throughput for automated cars (solid curves) under the *Baseline 'Weak' Scenario* and human-driven cars (dashed curves) per the HCM-2010 model (dashed lines; reproduced from TRB 2010, Exhibit 11-2 and 11-3). HCM-2010 specifies 45 passenger cars per lane per hour as the density at which a standard freeway segment is operating at its maximum capacity at all speeds.

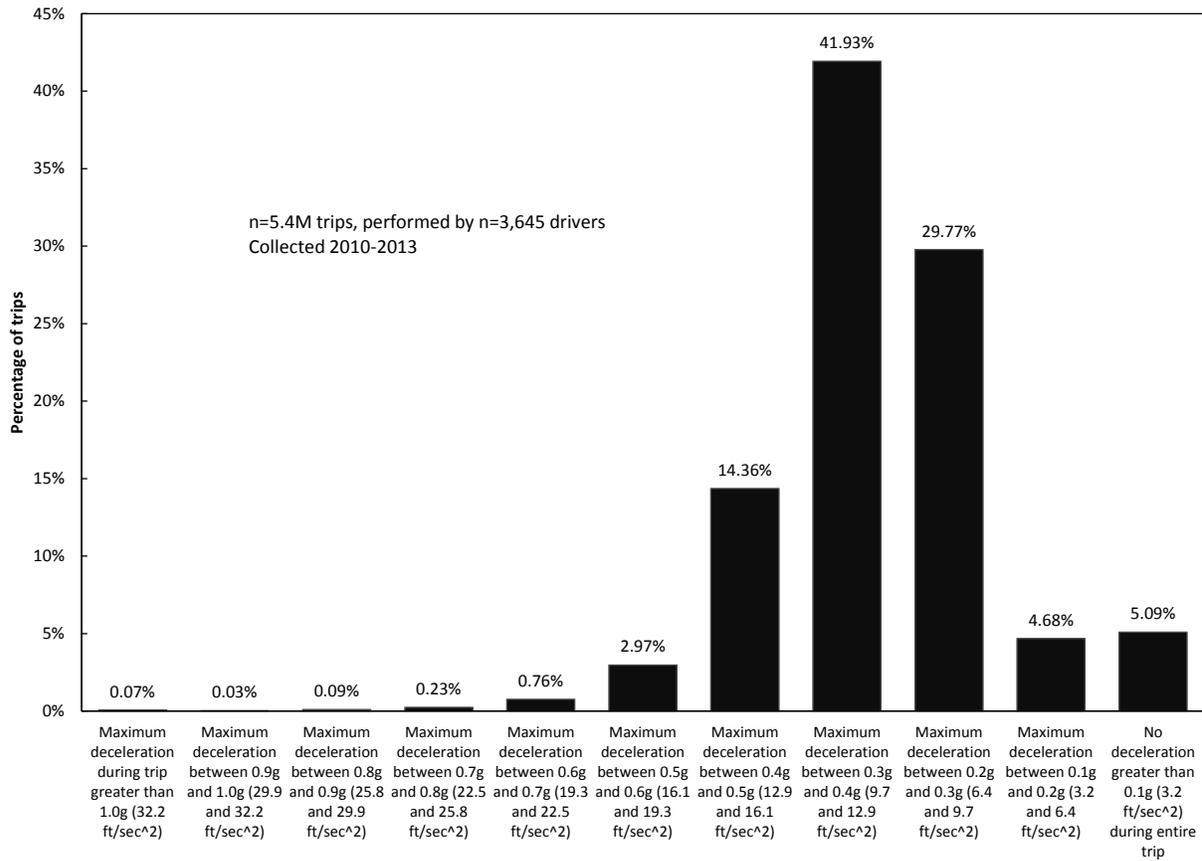


Figure 3: Distribution of journeys observed in the SHRP2 dataset (TRB 2013) by maximum rate of deceleration during each journey

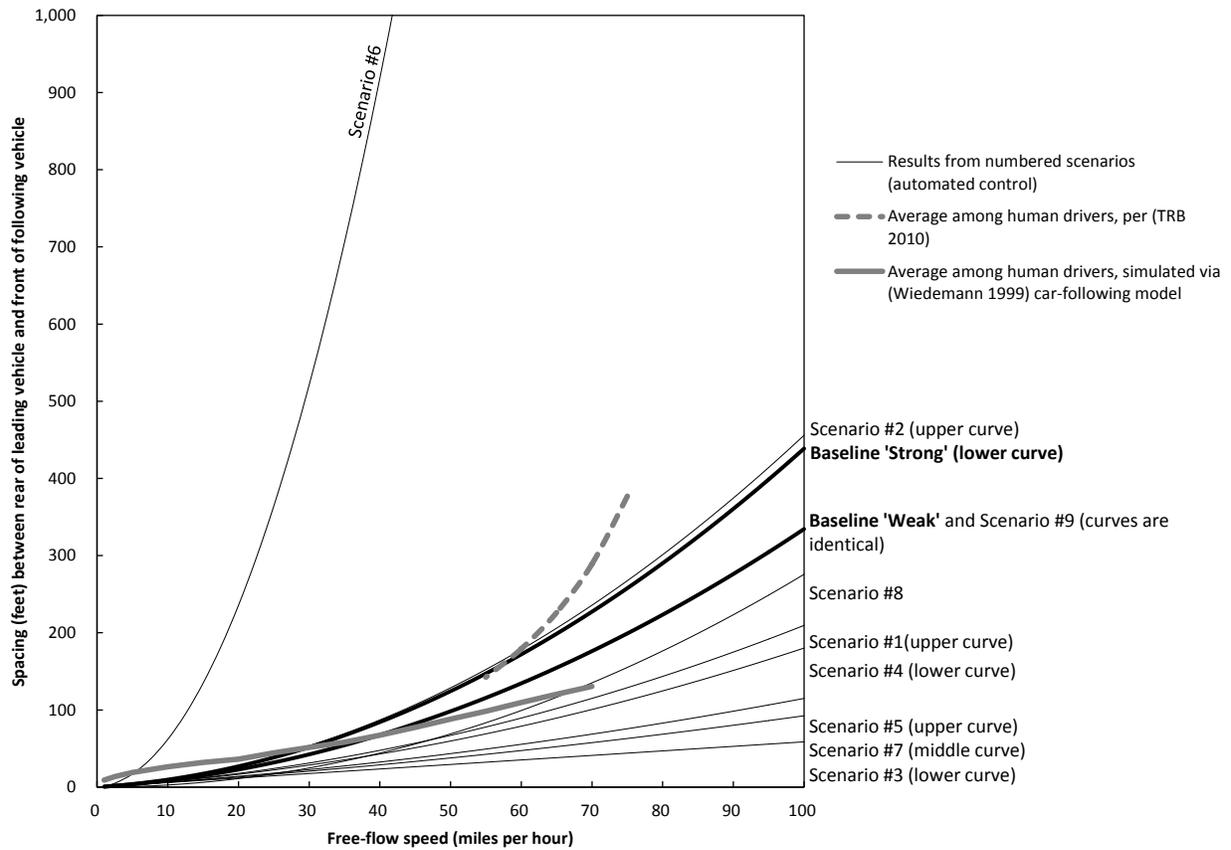


Figure 4: Calculations of spacing (rear of veh_l to front of veh_f) for automated vehicles (Baseline 'Weak' and 'Strong' Scenarios and Scenarios #1 through #9), with comparisons to human-driving behavior (HCM-2010 and Wiedemann-1999)

Table A1: Calculations of capacity (vehicles/lane-hour) for automated cars (Baseline and Scenarios #1 through #9), with comparisons to human-driving behavior (HCM-2010 and Wiedemann-1999). Table shows results from curves in Figure 1, in 5-mph increments.

Speed (mph)	Baseline 'Weak'	Baseline 'Strong'	Scenario #1	Scenario #2	Scenario #3	Scenario #4	Scenario #5	Scenario #6	Scenario #7	Scenario #8	Scenario #9	Human drivers (HCM-2010)	Human drivers (Wiedemann 1999)
5	1,167	1,154	1,183	1,152	1,204	1,187	1,196	735	1,199	1,341	964		696
10	1,911	1,842	2,002	1,831	2,123	2,024	2,076	653	2,095	2,427	1,631		1,164
15	2,329	2,179	2,539	2,156	2,849	2,594	2,725	515	2,773	3,142	2,044		1,555
20	2,528	2,299	2,872	2,265	3,436	2,967	3,202	415	3,291	3,516	2,270		1,912
25	2,593	2,299	3,063	2,257	3,921	3,199	3,550	344	3,689	3,643	2,372		2,080
30	2,579	2,237	3,157	2,189	4,328	3,332	3,802	293	3,996	3,615	2,394		2,247
35	2,521	2,147	3,185	2,095	4,675	3,396	3,980	255	4,232	3,501	2,367		2,392
40	2,439	2,045	3,171	1,992	4,973	3,412	4,103	225	4,412	3,346	2,312		2,447
45	2,347	1,942	3,128	1,888	5,233	3,394	4,183	202	4,548	3,175	2,242		2,460
50	2,251	1,842	3,068	1,788	5,462	3,354	4,231	182	4,650	3,002	2,163		2,463
55	2,156	1,747	2,997	1,694	5,664	3,299	4,252	167	4,723	2,835	2,083	1,800	2,473
60	2,064	1,658	2,919	1,606	5,845	3,235	4,255	153	4,774	2,678	2,002	1,600	2,462
65	1,976	1,576	2,839	1,525	6,007	3,164	4,242	142	4,806	2,533	1,924	1,400	2,465
70	1,893	1,501	2,758	1,451	6,153	3,090	4,217	132	4,824	2,398	1,849	1,200	2,470
75	1,816	1,431	2,678	1,382	6,286	3,015	4,184	123	4,829	2,274	1,777	1,000	
80	1,742	1,366	2,600	1,319	6,406	2,939	4,143	116	4,825	2,161	1,709		
85	1,674	1,307	2,523	1,261	6,517	2,865	4,098	109	4,812	2,056	1,645		
90	1,610	1,252	2,449	1,207	6,618	2,792	4,048	103	4,792	1,961	1,584		
95	1,550	1,201	2,378	1,158	6,712	2,720	3,996	98	4,767	1,872	1,528		
100	1,494	1,154	2,310	1,112	6,798	2,651	3,942	93	4,738	1,791	1,474		
Maximum throughput (capacity)	2,595	2,310	3,186	2,272	N/A	3,412	4,256	743	4,829	3,647	2,394	N/A	N/A
Speed (mph) at capacity	26.25	22.36	35.49	21.87	N/A	39.54	58.10	5.83	75.00	26.25	29.35	N/A	N/A

Table A2: Speed versus maximum throughput (vehicles per lane per hour) for human-driven cars per the HCM-2010 model (adapted from TRB 2010, Exhibit 11-2 and 11-3) and automated cars under the *Baseline 'Weak' Scenario*. Table shows results from curves in Figure 2, in 5-mph increments.

Speed (mph)	Automated cars (vehicles per lane per hour)	Human drivers on freeway with 75 mph (121 km/hour) free-flow speed	Human drivers on freeway with 70 mph (113 km/hour) free-flow speed	Human drivers on freeway with 65 mph (105 km/hour) free-flow speed	Human drivers on freeway with 60 mph (97 km/hour) free-flow speed	Human drivers on freeway with 55 mph (89 km/hour) free-flow speed
100	1,490					
95	1,549					
90	1,611					
85	1,675					
80	1,744					
75	1,815	1,000				
70	1,890	1,672	1,200			
65	1,976	1,950	1,856	1,400		
60	2,064	2,164	2,128	1,993	1,600	
55	2,156	2,344	2,337	2,239	2,124	1,800
50	2,250	2,250 ^a	2,250 ^a	2,250 ^a	2,250 ^a	2,250 ^a
45	2,349	2,025 ^a	2,025 ^a	2,025 ^a	2,025 ^a	2,025 ^a
40	2,440	1,800 ^a	1,800 ^a	1,800 ^a	1,800 ^a	1,800 ^a
35	2,520	1,575 ^a	1,575 ^a	1,575 ^a	1,575 ^a	1,575 ^a
30	2,580	1,350 ^a	1,350 ^a	1,350 ^a	1,350 ^a	1,350 ^a
25	2,592	1,125 ^a	1,125 ^a	1,125 ^a	1,125 ^a	1,125 ^a
20	2,528	900 ^a	900 ^a	900 ^a	900 ^a	900 ^a
15	2,329	675 ^a	675 ^a	675 ^a	675 ^a	675 ^a
10	1,910	450 ^a	450 ^a	450 ^a	450 ^a	450 ^a
5	1,167	225 ^a	225 ^a	225 ^a	225 ^a	225 ^a
Maximum throughput (speed at which this occurs in parentheses)	2,595 (at 26.25 mph [42.24 km/hour])	2,400 (at 53.30 mph [85.78 km/hour])	2,400 (at 53.30 mph [85.78 km/hour])	2,350 (at 52.20 mph [84.01 km/hour])	2,300 (at 51.10 mph [82.24 km/hour])	2,250 (at 50.00 mph [80.47 km/hour])

Notes: ^a indicates values sourced from the line of constant density of 45 vehicles per lane per mile. HCM-2010 specifies 45 passenger cars per lane per mile as the density at which a standard freeway segment is operating at its maximum capacity at all speeds.

Table A3: Calculations of spacing (rear of veh_l to front of veh_f , in feet) for automated vehicles (Baseline ‘Weak’ and ‘Strong’ and Scenarios #1 through #9), with comparisons to human-driving behavior (HCM-2010 and Wiedemann-1999). Table shows results from curves in Figure 4, in 5-mph increments.

Speed (mph)	Baseline ‘Weak’	Baseline ‘Strong’	Scenario #1	Scenario #2	Scenario #3	Scenario #4	Scenario #5	Scenario #6	Scenario #7	Scenario #8	Scenario #9	Human drivers (HCM-2010)	Human drivers (Wiedemann 1999)
5	4	4	3	4	3	3	3	17	3	1	4		19
10	9	10	7	10	6	7	6	62	6	3	9		26
15	15	17	12	18	9	12	10	135	10	6	15		32
20	23	27	18	28	12	17	14	236	13	11	23		36
25	32	38	24	39	15	22	18	364	17	17	32		44
30	42	52	31	53	18	29	23	521	21	25	42		51
35	54	67	39	69	21	35	27	706	25	34	54		58
40	68	84	48	87	23	43	32	919	29	44	68		67
45	82	103	57	107	26	51	38	1,159	33	56	82		78
50	98	124	67	129	29	60	43	1,428	38	69	98		88
55	116	147	78	152	32	69	49	1,725	42	83	116	142	98
60	134	172	90	178	35	79	55	2,049	47	99	134	179	110
65	155	199	102	206	38	89	62	2,402	52	117	155	226	120
70	176	227	115	236	41	101	69	2,783	58	135	176	289	131
75	199	258	129	267	44	112	76	3,191	63	155	199	377	
80	223	290	143	301	47	125	83	3,628	69	176	223		
85	249	324	159	337	50	138	91	4,092	74	199	249		
90	276	361	175	375	53	151	98	4,585	80	223	276		
95	305	399	192	414	56	165	107	5,105	86	249	305		
100	334	439	210	456	59	180	115	5,654	92	276	334		